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# Modelling of the catalytic combustion of lean fuel flowing over a fin with heat extraction from the trailing edge

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**Abstract**

The focus of this work was the study of catalytic combustion over a flat plate, or fin, from which heat was extracted from the trailing edge of the fin. This enabled those factors that effect the heat transfer to the sink to be elucidated. Experimental work was conducted that involved passing a heated lean fuel (either carbon monoxide or propane) in air mixture over a catalytic fin. The temperatures along the fin and across the fluid boundary layer were measured. Experimental conditions varied were bulk gas (or free stream) fuel concentration, coolant temperature, the temperature of the bulk gas and velocity of the bulk gas.

A model of the fin was derived using the basic equations of momentum, energy and conservation of chemical species for boundary layer flow. The model equations were solved numerically using a combination of orthogonal collocation and finite difference techniques and theoretical predictions were compared with experimental results. Theoretical predictions of the fin and boundary layer temperature were, generally, in excellent agreement with experimental values. 

*Keywords:* Catalysis; Combustion; Propane; Carbon monoxide; Platinum; Fin; Heat extraction; Boundary layer

## 1. Introduction

There can be significant advantages in utilising catalytic combustion instead of homogeneous based combustion. These are: (i) catalytic combustion can operate over a much wider range of fuel concentration and temperature conditions; (ii) the combustion products contain lower levels of pollutants in the formal of unburnt fuel, and $NO_x$, and (iii) there is *potential* for the efficient heat recovery from the combustion gases by conduction through the catalyst substrate. This last point is the most contentious and, to date, has received little attention.

Studies of catalytic combustion involving heat extraction have mainly focussed on the adiabatic operation of monoliths with energy extraction from the exhaust gases, such as gas turbines [1–8]. Relatively few studies have investigated the possibility of extracting combustion heat through the catalyst substrate. The metallic monolith, with its high surface area/volume ratio and good substrate heat transfer properties, has shown potential [9,10] for efficient heat extraction through the monolith material. Modelling of this process, however, is complicated due to complexity of the monolith geometry.

The focus of this work was to study catalytic combustion with heat extraction using a much simpler geometry; a flat plate, or fin, from which heat is extracted from the fin trailing edge. This enables those factors that effect the heat transfer to the sink to be elucidated. This work reports experimental measurement of the effect of bulk gas (or free stream) fuel concentration, coolant temperature, temperature of the bulk

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**Nomenclature**

| | |
|---|---|
| $A_c$ | pre-exponential constant in Arrhenius equation |
| $Bi$ | $hL/\lambda_p$ |
| $C_i$ | mass fraction of $i$th species (ppm by weight) |
| $C_P$ | heat capacity (J/kg K) |
| $d_p$ | half the thickness of fin (m) |
| $D_i$ | diffusivity of species $i$ |
| $Da(\mathrm{II})_i$ | $W_{p,i}L/\rho D_i$ |
| $Da(\mathrm{IV})$ | $\sum_i W_{p,i}\Delta h_i L/\lambda_f T_\infty$ |
| $h$ | heat transfer coefficient between cooling tube fluid and trailing fin edge |
| $\Delta h_i$ | heat of reaction for species $i$ in air (kJ/kg) |
| $l$ | $\rho\mu/\rho_\infty\mu_\infty$ |
| $L$ | fin length (m) |
| $Pr$ | $C_P\mu/\lambda_f$ |
| $Re_L$ | $\rho u_\infty L/\mu$ |
| $s$ | stream variable |
| $Sc_i$ | $\mu/\rho D_i$ |
| $St$ | $\alpha\sigma L T_\infty^3/\lambda_f$ |
| $T$ | temperature (K) |
| $u$ | velocity in $x$-direction (m/s) |
| $W_{p,i}$ | rate of formation of species $i$ at catalyst surface (kg/m$^2$ s) |
| $x$ | distance along fin (from leading edge) (m) |
| $x^*$ | dimensionless distance along fin |
| $y$ | distance perpendicular to fin surface (m) |

*Greek symbols*

| | |
|---|---|
| $\alpha$ | radiation heat transfer factor |
| $\eta$ | stream variable |
| $\theta$ | dimensionless temperature |
| $\lambda$ | thermal conductivity |
| $\mu$ | dynamic viscosity of gas (kg/m s) |
| $\rho$ | fluid density (kg/m$^3$) |
| $\sigma$ | Stefan–Boltzmann constant $= 5.67 \times 10^{-8}$ W/m$^2$ K$^4$ |
| $\varphi_i$ | dimensionless concentration of species $i$ |

*Subscripts and superscripts*

| | |
|---|---|
| 1 | propane |
| 2 | CO |
| f | fluid value |
| $i$ | referring to species $i$ |
| p | fin value |
| sink | sink value |
| $\infty$ | free stream value |
| $'$ | differentiation with respect to $\eta$ |
| $\cdot$ | differentiation with respect to $x^*$ |

gas and velocity of the bulk gas on the temperature profile of the fin. A model of the fin is derived and theoretical predictions compared with experimental results.

## 2. Experimental

The apparatus used was substantially the same as detailed in earlier publications [11,12]. A heated lean fuel (either carbon monoxide or propane) in air mixture was passed over a catalytic fin fixed within a flow channel. The catalytic fin was formed from a brass plate of dimension $l \times w \times d = 82\,\mathrm{mm} \times 72\,\mathrm{mm} \times 3.2\,\mathrm{mm}$ coated with a platinum/alumina catalyst which was brass soldered to a $\frac{1}{4}$ in. ID stainless steel tube along the trailing edge (Fig. 1). The fin was rounded at the leading edge. This was fixed in the reactor and was orientated such that the bulk flow of gases was parallel to the fin surface. The temperature of the fin was measured by five 1 mm diameter, type K thermocouples. These were inserted into the edge of the fin at five different distances along the fin. The thermocouples could be slid through thermocouple wells which transversed the width of the fin, thus the temperature profile across the fin width could also be measured. To measure the temperature in the gas above the fin surface, two thermocouples were placed at right angles to the fin surface at a distance of 52 and 80 mm from the fin leading edge.

Heat was extracted from the fin edge by cooler air flowing through the stainless steel tube. The rate of heat extraction from the fin to the coolant air was varied by changing the rate of air flow through the tube (and, hence, the resistance to heat transfer between the fin edge and the coolant air).

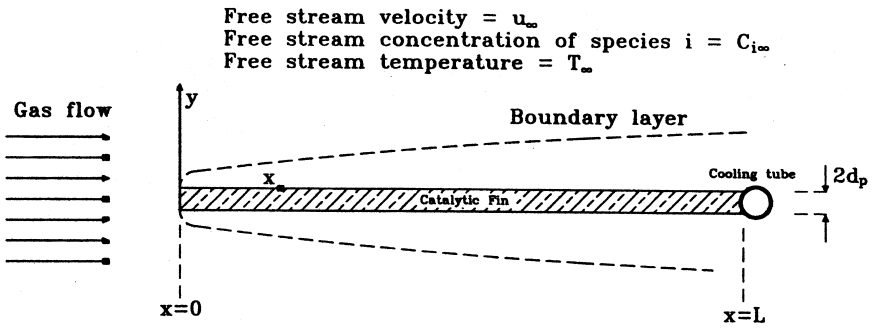


Fig. 1. Co-ordination system for catalytic fin.

## 3. Theoretical model

### *3.1. The equations of motion, energy and conservation*

The co-ordinate system for the model of the catalytic fin is shown in Fig. 1. Flow parallel to the fin induces the formation of a boundary layer. Fuel in the bulk gas is transported to the fin surface, where it reacts and generates heat. Heat may be transferred to or from the surrounding environment by convection and radiation. Heat may also be conducted through the fin substrate to the heat sink placed at its edge.

The equations of energy, motion and conservation are:

Equation of motion:

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{1}$$

Equation of energy:

$$\rho C_P\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial y}\left(\lambda_{\mathrm{f}}\frac{\partial T}{\partial y}\right) \tag{2}$$

Equation of conservation of the *i*th species

$$\rho C_P\left(u\frac{\partial C_i}{\partial x}+v\frac{\partial C_i}{\partial y}\right)=\frac{\partial}{\partial y}\left(\rho D_i\frac{\partial C_i}{\partial y}\right) \tag{3}$$

It is assumed that:

1. The bulk flow of gas is in the *x*-direction only and parallel to the fin surface.
2. Flow is laminar in the boundary layer — note all experiments reported were at Reynolds numbers within the laminar regime.
3. The gas is fuel lean — the bulk of the gas is composed of inert species.
4. Viscous dissipation is negligible.
5. Gravitational and buoyancy effects are negligible.
6. Conduction and diffusion in the *x*-direction are negligible.
7. Fluid and fin properties are constant.
8. Gas phase combustion is negligible.

The boundary conditions for the above equations are:

Motion boundary conditions:

$$\text{at } y=0: \quad u=v=0 \tag{4}$$

$$\text{as } y\to\infty: \quad u\to u_\infty \tag{5}$$

$$\text{at } x=0,\ y>0: \quad u=u_\infty,\ v=0 \tag{6}$$

Energy boundary conditions:

$$\text{at } y=0,\ 0<x<L: \quad -\lambda_{\mathrm{p}}d_{\mathrm{p}}\frac{\partial^2 T}{\partial x^2} = \lambda_{\mathrm{f}}\frac{\partial T}{\partial y}-\alpha\sigma(T^4-T_\infty^4)-W_{\mathrm{p},i}\Delta h_i \tag{7}$$

$$\text{at } y=0,\ x=0: \quad \frac{\partial T}{\partial x}=0 \tag{8}$$

$$\text{at } y=0,\ x=L: \quad -\lambda\frac{\partial T}{\partial x}=h(T-T_{\mathrm{sink}}) \tag{9}$$

$$\text{as } y\to\infty: \quad T\to T_\infty \tag{10}$$

The *i*th species concentration boundary conditions are:

$$\text{at } y=0: \quad \rho D_i\frac{\partial C_i}{\partial y}+W_{\mathrm{p},i}=0 \tag{11}$$

$$\text{as } y\to\infty: \quad C_i\to C_{i\infty} \tag{12}$$

$$\text{at } x = 0: \quad C_i = C_{i\infty} \tag{13}$$

### 3.2. Transformation of equations

The equations are non-dimensionalised using [12]

$$\eta = \frac{u_\infty}{\sqrt{s}} \int_0^y \rho \, \mathrm{d}y \tag{14}$$

$$s = (\rho\mu)_\infty u_\infty x \tag{15}$$

$$x^* = \frac{x}{L} \tag{16}$$

$$\theta = \frac{T - T_\infty}{T_\infty} \tag{17}$$

$$\phi_i = \frac{C_i - C_{i,\infty}}{C_{i,\infty}} \tag{18}$$

The resultant non-dimensional equations are:

Equation of motion:

$$2f''' + f''f = 0 \quad \text{(the Blasius equation)} \tag{19}$$

Equation of energy:

$$\frac{l}{Pr}\theta'' + \frac{1}{2} f\theta' = x^* f'\dot{\theta} \tag{20}$$

Equation of conservation of the *i*th species:

$$\frac{l\varphi_i''}{Sc_i} + \frac{1}{2} f\varphi_i' = x^* f'\dot{\varphi}_i \tag{21}$$

The boundary conditions for these equations are:

$$\text{at } \eta = 0, 0 \le x^* \le 1: \quad f = f' = 0 \tag{22}$$

$$\varphi_i' = -\frac{1}{C_{i\infty}} \sqrt{x^*} (Re_L l)^{-1/2} Da(\mathrm{II})_i \tag{23}$$

$$\text{at } \eta = 0, 0 < x^* < 1: \quad -\left(\frac{d_\mathrm{p}}{L}\right)\left(\frac{\lambda_\mathrm{p}}{\lambda_\mathrm{f}}\right)\ddot{\theta} = \sqrt{\frac{Re_L}{x^*}}\theta' - St((\theta+1)^4 - 1) - Da(\mathrm{IV}) \tag{24}$$

$$\text{at } \eta = 0, x^* = 1: \quad \frac{\partial\theta}{\partial x^*} = Bi(\theta_\mathrm{sink} - \theta) \tag{25}$$

$$\text{at } \eta = 0, x^* = 0: \quad \frac{\partial\theta}{\partial x^*} = 0 \tag{26}$$

$$\text{as } \eta \to \infty, 0 \le x^* \le 1: \quad \varphi_i = \theta = 0, f' = 1 \tag{27}$$

Table 1
Experimental conditions and parameters used in model

| Parameter | Symbol | Value |
|---|---|---|
| Free stream propane concentration | $C_{1\infty}$ | 0–15,000 ppm (w/w) |
| Free stream carbon monoxide concentration | $C_{2\infty}$ | 0–20,000 ppm (w/w) |
| Free stream velocity | $u_\infty$ | 1.0–2.0 m/s |
| Free stream temperature | $T_\infty$ | 473–773 K |
| Sink temperature | $T_\mathrm{sink}$ | 400–700 K |
| Fin length | $L$ | 82 mm |
| Half fin thickness | $d_\mathrm{p}$ | 1.59 mm |
| Radiation heat transfer factor | $\alpha$ | 0.47 |
| Fin thermal conductivity | $\lambda_\mathrm{p}$ | 0.14 kW/mK |

### 3.3. Solution method

The derived equations of energy and conservation were solved numerically using a combination of orthogonal collocation and finite difference techniques. Six collocation points were used in the $\eta$-direction. The generated non-linear equations were solved numerically using the IMSL finite difference routines DGEAR and DVCPR on a Cyber 990 computer.

The experimental conditions and model parameters used are listed in Table 1. The rate expressions for the catalytic reaction of the fuels are given in Table 2 [12]. The bulk properties of the gas used in the model were based on the properties of air.

## 4. Results and discussion

Fig. 2 shows a typical plot of the experimentally measured surface temperature at different distance across the fin. It can be seen that there is little variation in temperature across the fin, thus the process may be modelled in the two-dimensional domain. The figure also shows the increase in fin temperature that resulted from the addition of 20,000 ppm CO into the bulk airstream flowing over the fin.

To compare the theoretically predicted temperature profiles with the experimental data, it was necessary for the heat transfer coefficient between the fin edge/the cooling tube and the coolant fluid ($h$) to be known. This, however, was difficult to calculate so to generate the theoretical predictions, a temperature boundary condition at $x = 1$ was used based on

Table 2
Reactions and rate expressions used [12]

| Reaction | Rate expression |
|---|---|
| $C_3H_8 + 5O_2 \overset{Pt/Al_2O_3}{\rightarrow} 3CO_2 + 4H_2O$ | $W_{p,1} = -A_c \exp\left[\frac{-E_a}{RT}\right] \rho^{1.5} C_1^{1.5}$ kg/m$^2$ s, $\ln A_c = 5.98 \pm 1.99$, $E_a = 49.6 \pm 12.5$ kJ/mol |
| $CO + \frac{1}{2}O_2 \overset{Pt/Al_2O_3}{\rightarrow} CO_2$ | $W_{p,2} = \frac{-A_c \exp[-E_a/RT]}{\rho C_2}$ kg/m$^2$ s, $\ln A_c = 7.65 \pm 1.49$, $E_a = 82.4 \pm 10.4$ kJ/mol |

extrapolating the experimental surface temperatures to the trailing edge.

Figs. 3–7 show the effect of free stream fuel concentration, free stream velocity, free stream temperature and temperature at the trailing edge ($\theta_{sink}$) on the temperature profile along the fin. Better agreement between experimental data and theoretical predictions was observed for combustion using CO than with propane. Under reacting conditions, the theoretical predictions indicated the reaction of CO was mass transfer controlled, whereas the reaction of propane was neither mass transfer nor kinetically controlled. Hence, the theoretical predictions for propane were more sensitive to the reaction rate equations adopted than CO. Previous research conducted on a fin operated adiabatically [12] indicated that errors in the experimentally derived reaction rate equations used for propane combustion could account for the discrepancy between theoretical and experimental results.

Figs. 3 and 4 show that increasing the free stream fuel concentration resulted in a greater driving force for mass transfer of fuel to the fin surface and, hence, increased surface reaction and heat generation resulting in a rise in fin temperature.

Increasing the free stream velocity (Fig. 5) results in a decrease in boundary thickness with consequent increase in mass transfer of fuel to the fin surface. Thus the rate of surface reaction, and heat generation, was

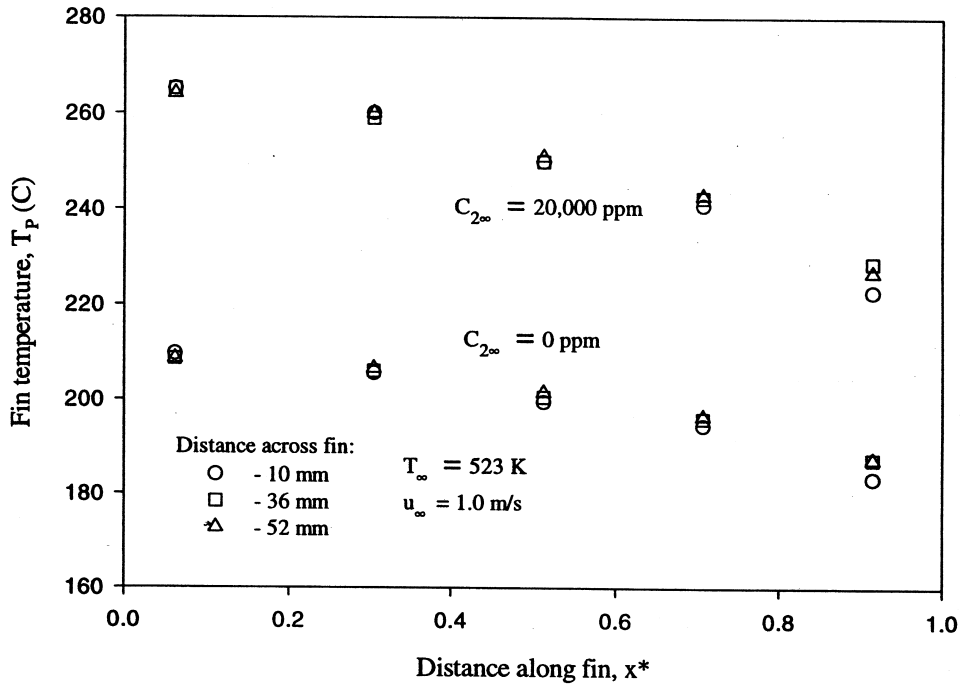


Fig. 2. Typical temperature profile along and across fin.

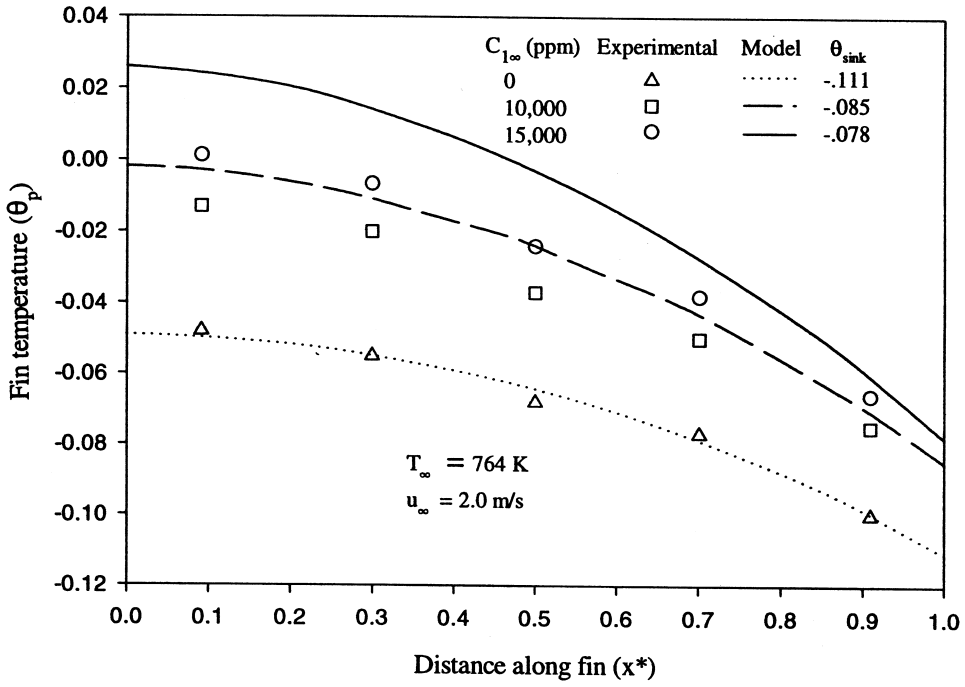


Fig. 3. Effect of propane concentration on fin temperature.

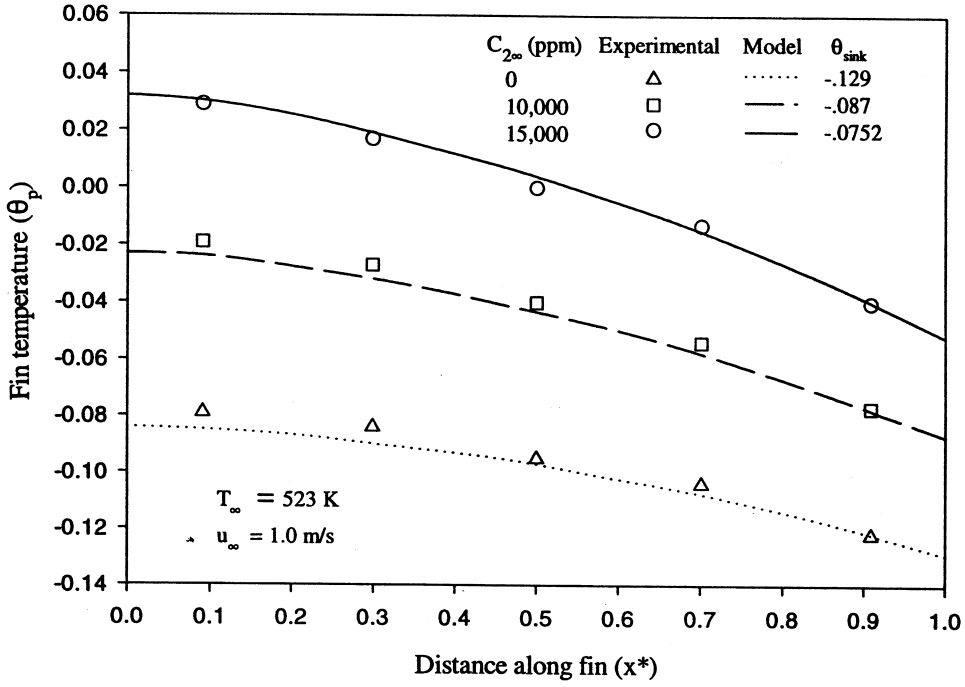


Fig. 4. Effect of CO concentration on fin temperature.

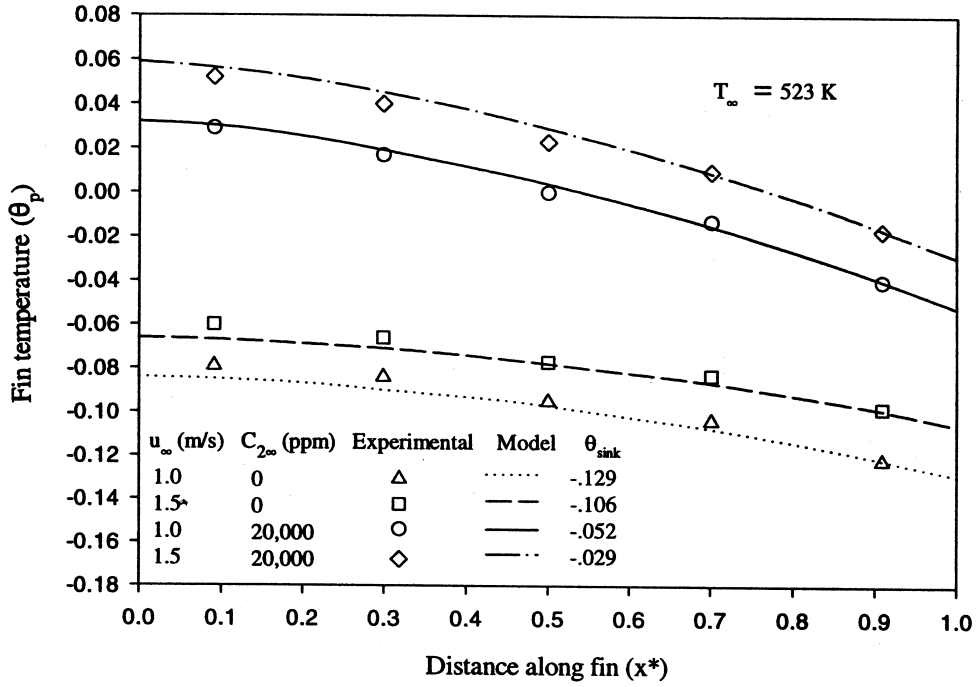


Fig. 5. Effect of free stream velocity on fin temperature.

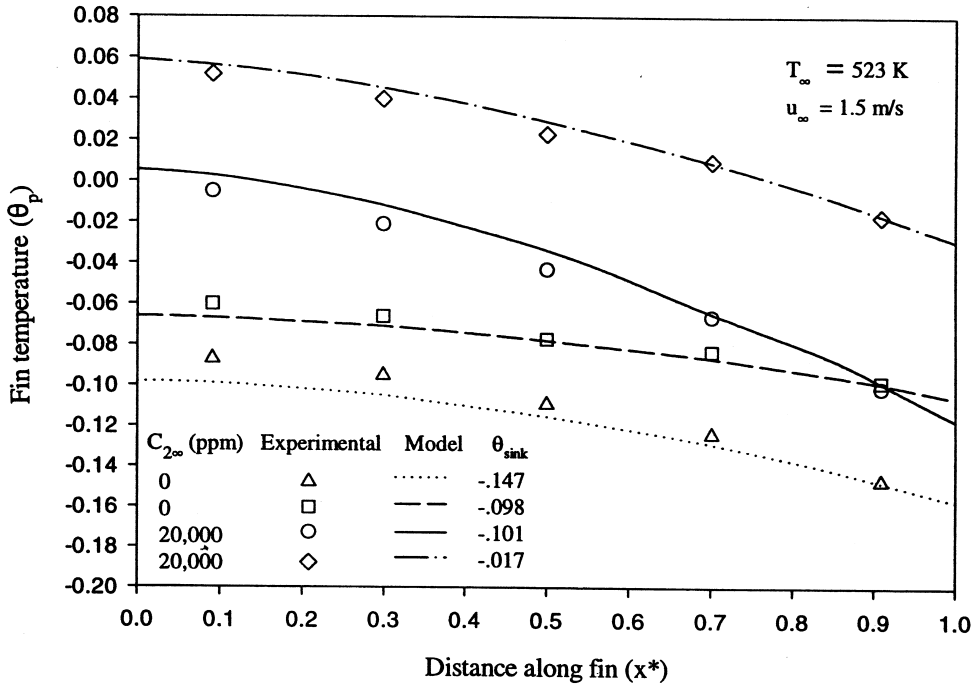


Fig. 6. Effect of trailing edge temperature on fin temperature.

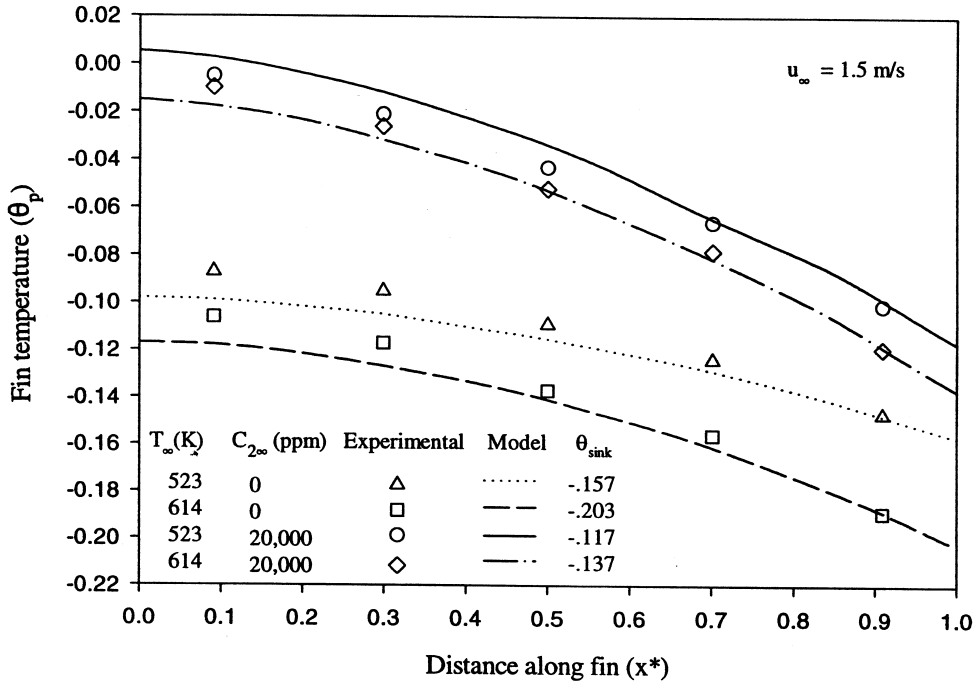


Fig. 7. Effect of free stream temperature on fin temperature.

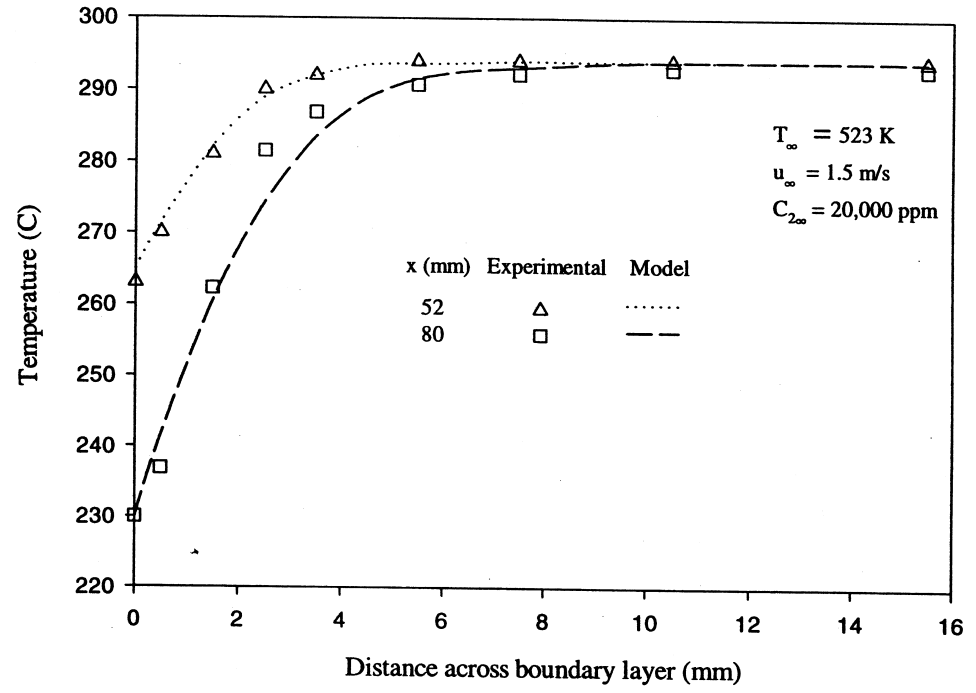


Fig. 8. Temperature profile across boundary layer.

increased. As the cooling pipe was not insulated from the flowing bulk gases, increasing the free stream velocity also had the effect of increasing the heat transfer of gas to the cooling tube. This resulted in an increase in cooling fluid temperature. The combined effect of increased coolant fluid temperature and surface reaction resulted in an increase in fin temperature.

As expected, increasing the coolant temperature (Fig. 6) resulted in an increase in fin temperature. Similarly, increasing the free stream temperature (Fig. 7) increased the fin temperature. The effect was greater with propane fuel as the fin was not operating in the mass transfer controlled regime.

Fig. 8 shows a typical temperature profiles across the boundary layer. There is good agreement between the predicted and experimental data. The increase in boundary layer thickness is evident at increased distance along the fin.

## 5. Conclusions

Experimental and theoretical studies were made of the combustion of CO and propane in air over a catalytic fin. The theoretical predictions of the effect of free stream fuel concentration, free stream velocity, free stream temperature and fin edge temperature on the temperature profile of the fin and the temperature profile across the induced boundary layer agreed well.

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